

# Correlations in Frequency Comparison Data

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The number of high-accuracy frequency comparisons involving optical atomic clocks has increased significantly in recent years, as the international community strives to meet the criteria necessary for a redefinition of the second in the International System of Units (SI)<sup>1</sup>. Such comparisons are vital to verify the uncertainty budgets of optical clocks developed independently in different laboratories around the world. The results of such comparisons are also used by the Frequency Standards Working Group (WGFS) of the Consultative Committee for Time and Frequency (CCTF) and the Consultative Committee for Length (CCL) to derive recommended frequency values and uncertainties for secondary representations of the second. These recommended frequency values and uncertainties are used when optical clocks contribute data to the International Bureau of Weights and Measures (BIPM) for steering of International Atomic Time (TAI).

The most recent update to the recommended frequency values, performed in 2021, differed from earlier updates in that correlations between frequency ratio measurements in the input dataset were computed and taken into account for the first time<sup>2</sup>. Such correlations are becoming increasingly significant as more and more comparisons are performed in large-scale measurement campaigns involving multiple institutions, and as the uncertainties of the frequency ratio measurements are reduced. Neglecting correlations would result in biased values for the recommended frequencies as well as incorrectly estimated uncertainties<sup>3</sup>. The change in approach was influenced by our guidelines<sup>4</sup> developed as part of the EMPIR-funded project “Robust Optical Clocks for International Timescales” (ROCIT).

We will discuss the various ways in which correlations between frequency ratio measurements may arise and describe how they can be quantified. The most significant correlations usually originate from performing multiple comparisons in the same measurement campaign or from using common primary or secondary frequency standards as a reference. The latter source of correlation must be considered not only for measurements made against local standards but also for measurements performed via a frequency link to TAI. We hope that this discussion will encourage groups submitting data to the WGFS to compute and communicate correlation coefficients themselves, in particular for measurements performed within their own institutes, or for measurements performed within a coordinated comparison campaign. This should help ease the work of the WGFS in the next update to the list of recommended frequency values, which is expected in 2025.

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<sup>1</sup> N. Dimarcq *et al.*, “Roadmap towards the redefinition of the second”, *Metrologia*, vol. 61, 012001, 2024.

<sup>2</sup> H. S. Margolis, G. Panfilò, G. Petit, C. Oates, T. Ido and S. Bize, “The CIPM list ‘Recommended values of standard frequencies’: 2021 update”, arXiv:2401.14537, 2024.

<sup>3</sup> H. S. Margolis and P. Gill, “Least-squares analysis of clock frequency comparison data to deduce optimized frequency and frequency ratio values”, *Metrologia*, vol. 52, p. 628–634, 2015.

<sup>4</sup> H. S. Margolis and M. Pizzocaro, “Guidelines on the evaluation and reporting of correlation coefficients between frequency ratio measurements”, <https://empir.npl.co.uk/rocit/project-outputs/>, 2020.